range of γ_1. The boom is unstable (if the damping is negligible) for any $\gamma_1 < 0$ (or $\sin \alpha < 0$), i.e., "if pointed away from the sun."

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Comment on "Thermally Induced Vibration and Flutter of a Flexible Boom"

PETER F. JORDAN*

RIAS, Martin Marietta Corp., Baltimore, Md.

THERE is a fallacious argument in the paper on thermal bending flutter of long flexible booms by Yi-Yuan Yu.[1] In consequence, the main result[1] is just the reverse of the correct answer.

Thermal flutter is readily accessible to physical intuition. We will clarify this statement first, and will then identify the error in Ref. 1.

From the point of view of the theory of dynamic stability,† the thermal flutter of booms corresponds to the aerodynamic flutter of airplane wings. In both, there is an elastic structure imbedded in a continuous stream of energy (of flowing air or of thermal radiation). There are oscillatory reactions from the stream onto the structure, which arise when the structure undergoes some oscillatory deformation (started by some incidental disturbance).

The mechanism which most often causes aerodynamic flutter is coalescence of the frequencies of two different degrees of freedom. However, large reaction forces are required to make two frequencies coalesce, and the thermal reactions which occur in space may be relatively weak. It thus makes sense to look at the case of a boom which oscillates in single degrees-of-freedom bending and is exposed to low-intensity thermal radiation. In essence, this is what Yu[1] sets out to do.

Whether the effect of the thermal field will be stabilizing ("damping"; oscillatory structural energy is lost) or destabilizing ("flutter"; oscillatory energy is extracted by the structure from the continuous field of thermal energy) depends upon the product of two signs, 1) the direction of the reactive effects, and 2) the phase (time) relation between reactive effect and structural oscillation.‡

Here one can refer to the familiar analytical concept of structural damping. The structural damping reaction has the same direction as the elastic reaction (structural damping is "restoring") but it is *advanced* in phase. Hence, we know that this combination of the two signs causes loss of energy, damping of the oscillatory motion.

The thermal reaction is *delayed* in phase. Thus we come to a simple rule R for thermal dynamic stability:

"A 'restoring' thermal reaction (one which is stabilizing in the static situation) will tend to produce flutter (will be destabilizing in the dynamic situation), and vice versa."

Applied to the problem of thermal bending flutter, R predicts: 1) the boom will be prone to flutter if it is directed away from the sun (in this case the thermal effect is restoring: to deflect the boom will cause a change in the thermal input such that the deflection is reduced); and 2) the boom will not flutter if it is directed toward the sun (in this case there is a tendency towards static thermal divergence).

These predictions regarding thermal bending flutter agree with the analytical results of G. Augusti.[4]§ Applied to the problem of thermal torsional flutter of open booms, R predicts flutter if the thermal radiation is directed toward the slit. This prediction is beautifully confirmed by the experiments of R. M. Beam.[7]

In contrast to R, Yu[1] states: "(Bending) motion is stable if the boom is pointed away from the sun and unstable if towards the sun." We next identify the fallacy in the analysis of Ref. 1 which led Yu to his erroneous statement.

We start with the here required terms of Eq. (2) or Eq. (3) of Ref. 1. These terms are listed on the right-hand side of the following relation:

$$\int_0^l M_T w'' dx = \int_0^l M_T'' w dx + [M_T w' - M_T' w]_0^l \quad (1)$$

Details which are here unnecessary have been left out in Eq. (1).

The integral on the left hand side of Eq. (1) arises when one writes the work done by the thermal moment M_T, namely, its product with the change in the beam curvature, w''. The right-hand side is the result of integrating twice by parts.

From Eqs. (5) and (7) of Ref. 1

$$M_T = EI[-k_c + k_s(w' - \tau \dot{w}')] \quad (2)$$

Since we are only concerned with the question of stability, we are interested only in out-of-phase terms, and can disregard k_c. For the remainder we use the following as a tracer

$$k_s w' \text{ traces } M_T \quad (3)$$

The tracer terms in Eq. (1) are hence

$$k_s \int_0^l w' w'' dx = k_s \left[\int_0^l w''' w dx + (w'^2 - w w'')_0^l \right] \quad (4)$$

The terms on the right-hand side of Eq. (4) should appear in Eq. (10) of Ref. 1. However, having informed the reader that to leave out the end terms "simplifies greatly," Yu retains only the integral; thus

$$\left(\int_0^l M_T w'' dx \right)_{\text{tracer}} = k_s \int_0^l w' w'' dx \overset{?}{=} k_s \int_0^l w''' w dx\text{¶} \quad (5)$$

Let us see how well this simplification is justified.

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* Principal Research Scientist. Associate Fellow AIAA.

† "Dynamic stability" is here used in its natural meaning, N. J. Hoff,[2] rather than in the restrictive sense of V. V. Bolotin.

‡ In 1929, H. Glauert[3] showed that energy extraction is possible in principle by showing that single degree-of-freedom flutter of airplane wings is a physical possibility even in the fully linearized formulation of the aerodynamic problem.

§ The writer much regrets that he was unaware of this pioneering work[4] when he wrote his own paper, Ref. 5 and welcomes the fact that Professor Augusti has now undertaken[6] to familiarize the readers of the *Journal of Spacecraft and Rockets* with his results.

¶ Yu,[1] Eq. (10).

First we set $w = x^2/l$ (the familiar rough approximation for the fundamental bending mode). Then Eq. (4) reads

$$k_s \cdot 2 = k_s[0 + (4 - 2)] \tag{6}$$

This example does not support the Yu contention that the end terms may be left out. Of course, we used a poor approximation; let us try a better one. If the thermal field is weak, then the bending mode will be, almost, a natural bending mode w_n, and this we now assume

$$w(x) \approx w_n(x) \tag{7}$$

Using the boundary conditions

$$w_n(0) = w_n'(0) = w_n''(l) = w_n'''(l) = 0 \tag{8}$$

we find that Eq. (4) becomes

$$k_s\left[\frac{1}{2}\int_0^l (w_n'^2)'dx\right] = k_s\left\{\int_0^l w_n'''w_n dx + [w_n'^2(l) - 0]\right\}$$

whence

$$k_s[\tfrac{1}{2}\, w_n'^2(l)] = k_s\{-\tfrac{1}{2}\, w_n'^2(l) + [w_n'^2(l)]\} \tag{9}$$

Here the "simplification" Eq. (5) amounts to a straightforward reversal of the sign of the k_s term.

Now the term with k_s in Eq. (10) of Ref. 1 is the one term which matters for stability. This term is described by Eq. (9) to the degree that the approximation Eq. (7) is valid, and of this approximation Yu makes use later in his paper.** This fully explains why the Yu result is just the reverse of the correct answer.

Certain misunderstandings remain to be clarified which have arisen in connection with the original version of Ref. 1.[8] This version contains considerable matter which, as a patient reader would find out eventually, is not used in the actual analysis. It also contains some strange remarks.†† A hurried reader, let us call him B, does not readily find out what is actually being done. He prefers to make his independent analysis.

The point to be made here is twofold: 1) the two analytical approaches, the one chosen by Yu and the one chosen by B, are superficially similar, and, by coincidence, their results also look deceptively similar; and 2) there is in fact a very real difference.

B starts with the left-hand side of Eq. (1) (so does Ref. 8) and derives Eq. (2) for himself. Realizing that M_T, though written as a moment, is in fact a thermally induced curvature which does not involve structural forces, B does not try to transform M_T into a "force" by means of the double integration by parts in Eq. (1). He goes the direct way, writing

$$\left(\int_0^l M_T w'' dx\right)_{\text{tracer}} = k_s \int_0^l w'w''dx = k_s\left(ww''|_0^l - \int_0^l ww'''dx\right) \tag{10}$$

This result, the right-hand side of Eq. (10), B compares with the Yu result, the right-hand side of Eq. (5).

B sees certain end terms in Eq. (10) which are not in Eq. (5). But, and this is the point, these end terms are unlike those in Eq. (4). The end terms in Eq. (10) disappear, all of them, when the natural approximation Eqs. (7) and (8) is introduced. B readily accepts that to leave out the end terms "simplifies greatly."

** Note that Eq. (9) implies that w_n''' is essentially negative. This fact Yu does notice (in the paragraph after Table 1) but he does not heed the warning which this fact implies.

†† Much of this is eliminated in Ref. 1. Still in Ref. 1 is the following remark: ". . . we do not consider booms which are nearly parallel to the sun ray." This remark eliminates, for no reason which this writer can see, just that angular range where instability is most pronounced (including the case, boom and ray parallel, which corresponds to the analysis of Augusti[4]). The author should explain.

This leaves B with the case of the missing minus sign. Not ready to suspect an error in an elementary analysis, B is ready to be persuaded that there is a wrong sign with the physical constant k_s. After all, action and reaction do get mixed up sometimes.

In the here relevant section II of his paper[5] for the ASME/AIAA SDM Conference (which section was a last minute addition) this writer, one of several B's, committed two sins for which he now wishes to apologize: 1) he reported his derivation (of a correct result) in a hasty manner which he does not wish to defend in detail; and 2) he stated that the sign error[8] was in the constant k_s.

At the Conference, J. D. Graham stated privately that he suspected the error source to be in a boundary condition. After the Conference, the error source was properly identified, and the subsequent letter exchange showed agreement with Graham.[9]

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Reply by Author to P. F. Jordan and G. Augusti and New Results of Two-Mode Approximation Based on a Rigorous Analysis of Thermal Bending Flutter of a Flexible Boom

Yi-Yuan Yu*

Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio

Some questions have been raised since the author's original paper and its preprint form were published.[1] For the benefit of those interested in the problem of thermal bending flutter, a summary of their current status seems to be in or-

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* Professor and Assistant for Advanced Research in Aerothermomechanics; formerly Consultant, General Electric Company, Valley Forge, Pa. Associate Fellow AIAA.